



Time-Reversal Symmetry Breaking in Condensed Matter Systems

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Abstract:

Time-reversal symmetry breaking (TRSB) is a key phenomenon observed in various condensed matter systems, influencing the behavior of quantum materials. This paper explores TRSB in different systems such as superconductors, magnetically ordered materials, and topological insulators, focusing on its implications for exotic quantum states and new materials design. The article discusses experimental evidence and theoretical frameworks, providing a comprehensive overview of how TRSB affects the properties of these materials, including magnetization, charge transport, and superconductivity. Special attention is given to the detection methods, including non-invasive techniques that highlight TRSB in complex systems.

Keywords: *Time-reversal symmetry, superconductivity, quantum materials, condensed matter, magnetism, topological insulators, TRSB detection, quantum states.*

Introduction:

Time-reversal symmetry is a fundamental concept in physics, referring to the invariance of physical laws under the reversal of time. In condensed matter systems, TRSB signifies a break in this symmetry, which can lead to the emergence of novel quantum phases and topologically non-trivial states. TRSB is significant in understanding the behavior of materials that exhibit unconventional superconductivity, magnetic ordering, and quantum phase transitions. This paper examines the various mechanisms that lead to TRSB in condensed matter systems and their implications for both theoretical models and experimental observations.

1. Fundamental Concepts of Time-Reversal Symmetry and its Breaking:

Overview of Time-Reversal Symmetry in Physics:

Time-reversal symmetry (TRS) is one of the fundamental symmetries in physics, describing the invariance of physical laws when the direction of time is reversed. In classical mechanics, time-reversal symmetry implies that if the velocities of all particles in a system are reversed, the system's evolution will be indistinguishable from the original state, provided no other factors influence the dynamics. In quantum mechanics, the situation is similar, but it involves the reversal of the time

operator on wavefunctions. TRS plays a crucial role in both microscopic and macroscopic systems, dictating how systems behave under the transformation of time reversal.

In quantum mechanics, time-reversal symmetry is associated with an anti-unitary operator T , which, when applied to the system's Hamiltonian, leaves the physical laws unchanged. However, certain quantum states, such as those exhibiting magnetization or spin currents, may break time-reversal symmetry, leading to interesting physical phenomena.

Mathematical Formulations of Time-Reversal Symmetry:

The mathematical formulation of time-reversal symmetry depends on the context of the system. In quantum mechanics, the time-reversal operator T is anti-unitary, which means that it involves both a complex conjugation and a reversal of time. For a state $|\psi(t)\rangle$, time-reversal symmetry implies that:

$$T|\psi(t)\rangle = |\psi(-t)\rangle, \quad \langle\psi(t)| = \langle\psi(-t)|T$$

In the Hamiltonian formalism, the time-reversal operator affects the time evolution of a quantum state by reversing the momentum and spin while complex-conjugating the coefficients. In terms of mathematical structure, time-reversal symmetry can be expressed as:

$$THT^{-1} = H$$

This equation states that the Hamiltonian H , which governs the system's dynamics, remains unchanged under the time-reversal operation. For systems exhibiting time-reversal symmetry, the corresponding eigenvalues and eigenstates are symmetric under time reversal.

Conditions Under Which TRSB Occurs in Condensed Matter Systems:

Time-reversal symmetry breaking (TRSB) occurs when the physical system does not remain invariant under the reversal of time. In condensed matter systems, TRSB is often associated with phenomena such as magnetization, spin currents, and unconventional superconductivity. Several conditions can lead to TRSB in these systems:

Magnetism: In systems where spontaneous magnetization occurs, time-reversal symmetry is inherently broken because the magnetization points in a particular direction, which is not time-symmetric. In ferromagnets, for instance, the spin orientation of the electrons in the material favors a specific direction, and reversing time would invert the direction of magnetization.

Superconductivity: In unconventional superconductors, such as those exhibiting p-wave pairing, TRSB is observed. These superconductors break TRS due to the nature of the pairing mechanism between electrons. For example, in systems where Majorana fermions are predicted to emerge, TRSB plays a key role in the formation of these topologically non-trivial states.

Topological Phases: In certain topological materials, TRSB leads to the appearance of exotic states, like the quantum Hall effect or the surface states of topological insulators. These materials exhibit robust edge states that are protected by symmetry and are inherently non-trivial under time-reversal operations.

Spin-Orbit Coupling: Systems with strong spin-orbit coupling can exhibit TRSB. This occurs because the spin of the electrons couples to their momentum in such a way that time reversal causes an irreversible interaction. This interaction can break TRS, resulting in phenomena like the spin Hall effect, where the electron's spin is separated from its charge.

External Fields: When external fields, such as magnetic or electric fields, are applied to a system, they can break time-reversal symmetry. For example, the application of a magnetic field in a material induces a preferential direction for the spins, thereby violating TRS. Similarly, in systems under intense electric fields, the induced polarization or charge separation may also lead to TRSB. In conclusion, TRSB is a critical feature in condensed matter physics, influencing the electronic, magnetic, and superconducting properties of materials. Understanding the conditions under which

TRSB occurs helps in the design of materials for next-generation technologies like quantum computing, spintronics, and energy-efficient superconductors.

2. TRSB in Superconductors:

The Role of TRSB in Unconventional Superconductivity:

Time-reversal symmetry breaking (TRSB) plays a significant role in unconventional superconductivity, particularly in systems where the pairing mechanism between electrons does not follow the conventional s-wave symmetry. In conventional superconductors, electrons pair up in a state that is symmetric under time-reversal; this symmetry protects the Cooper pairs from any changes in the system's evolution when time is reversed. However, in unconventional superconductors, TRSB can arise due to the nature of the electron pairing.

In these systems, such as in p-wave or d-wave superconductors, the pairing between electrons involves more complex spatial symmetries, breaking the time-reversal symmetry. TRSB is essential in these materials because it is associated with the formation of topologically non-trivial states, including Majorana fermions in certain superconductors. In these systems, the TRSB leads to the emergence of exotic quantum states that could potentially be harnessed for quantum computing and other advanced technologies.

TRSB in unconventional superconductors also influences the magnetic properties of the superconducting state. For example, it can result in the creation of spontaneous magnetic moments or vortices in the superconductor. This is particularly significant in high-temperature superconductors (HTS), where the interplay between superconductivity and magnetism can lead to unique phenomena, such as the emergence of topologically protected states or the formation of exotic quasiparticles.

Theoretical Models: p-wave Superconductors and Majorana Fermions:

Theoretical models have been crucial in understanding how TRSB affects superconductivity. In p-wave superconductors, the Cooper pairs are formed through an interaction that has a spatially anisotropic wavefunction, meaning that the pairing symmetry is different from the conventional s-wave pairing in regular superconductors. The time-reversal symmetry in p-wave superconductors is broken because the wavefunction of the paired electrons has an angular dependence, and flipping the time changes the direction of the electron spins. This leads to the formation of a time-reversed invariant state that is topologically distinct from conventional superconductivity.

One of the most exciting aspects of p-wave superconductivity is the possibility of Majorana fermions, which are predicted to emerge in systems that exhibit TRSB. Majorana fermions are quasiparticles that are their own antiparticles, and they are predicted to have significant applications in quantum computing. In these systems, the TRSB is essential for the protection and stability of these Majorana modes, as they arise in the vortex cores or at the boundaries of the superconductor, where TRSB and spin-orbit coupling play a crucial role.

Experimental Evidence of TRSB in High-Temperature Superconductors:

High-temperature superconductors (HTS), particularly those based on copper-oxide (cuprate) and iron-based materials, provide compelling experimental evidence of TRSB in unconventional superconductivity. In these materials, TRSB has been observed in several experimental setups, providing critical insight into their unique electronic structure.

One of the most well-known experimental techniques for probing TRSB in HTS is the muon spin rotation (μ SR) method, which has been used to detect the presence of magnetic fields in the superconducting state. In cuprates, μ SR experiments have shown that the superconducting state in the presence of a magnetic field can exhibit TRSB, manifesting in the form of spontaneous

magnetic moments or currents that violate the symmetry of the system. This is particularly relevant in cuprates, where the competition between superconductivity and magnetism leads to the formation of exotic phases that break TRSB.

Another technique used to investigate TRSB in HTS is the scanning tunneling microscopy (STM), which allows for the detailed study of the electronic structure of the superconducting state. In iron-based superconductors, STM measurements have revealed the presence of nodal and antinodal gaps, suggesting that the superconducting pairing is unconventional and may involve a symmetry-breaking mechanism that includes TRSB.

Furthermore, the discovery of the "unusual" magnetic order in some high-temperature superconductors, such as the "striped" phases in cuprates, is another indication of TRSB. These phases are characterized by periodic magnetic fluctuations, which break the time-reversal symmetry and result in novel quantum phases. This breaking of TRS is linked to the underlying electron pairing mechanism in these materials, which contributes to their high-temperature superconducting properties.

In summary, TRSB in superconductors is fundamental to understanding the behavior of unconventional superconductors. Through theoretical models and experimental evidence, it is clear that TRSB influences the pairing mechanism, stability, and the formation of topologically protected states, including Majorana fermions. The presence of TRSB in high-temperature superconductors reveals the complex interplay between superconductivity and magnetism, opening new avenues for future materials and quantum technologies.

3. TRSB in Magnetic Materials:

Spintronics and the Relationship Between TRSB and Magnetization:

Spintronics, or spin electronics, is a field of study focused on utilizing the intrinsic spin of electrons, along with their charge, to develop new types of devices. The relationship between time-reversal symmetry breaking (TRSB) and magnetization is central to spintronic devices. Magnetization is a manifestation of the alignment of the spins of electrons in a material, and it naturally breaks time-reversal symmetry because the magnetization points in a specific direction. In simple terms, if time were reversed, the direction of magnetization would flip, making it a non-symmetric process.

TRSB plays a significant role in spintronic devices, as it leads to the creation of asymmetric spin distributions. For instance, in systems with spin-polarized currents, such as magnetic tunnel junctions (MTJs), TRSB can influence the efficiency of charge and spin transport, allowing for devices that leverage the spin degree of freedom. The breaking of TRS also enhances the stability and robustness of certain spintronic states, such as in the case of the topological spin states in materials like spin Hall insulators or the spin-orbit torque mechanism in ferromagnetic materials. These spintronic phenomena rely heavily on TRSB to enable the controlled manipulation of spin currents for use in memory and logic devices.

TRSB in Ferromagnets and Antiferromagnets:

In ferromagnetic materials, the magnetization arises from the parallel alignment of electron spins in the same direction. This spontaneous magnetization leads to TRSB because the system's magnetic moment defines a preferred direction in space, which cannot be reversed by simply flipping the time direction. In ferromagnets, TRSB is often associated with the macroscopic magnetization of the material, making these systems excellent candidates for TRSB-based phenomena in spintronics. The presence of a stable magnetic moment in ferromagnetic materials

results in non-reciprocal effects such as the anomalous Hall effect (AHE) and the magnetoresistance (MR), both of which are sensitive to the time-reversal symmetry breaking.

In contrast, antiferromagnetic materials exhibit a different type of magnetization, where adjacent spins are aligned in opposite directions, resulting in a net zero macroscopic magnetization. However, TRSB can still occur in antiferromagnets, particularly when the spins in the system are coupled in a non-collinear or complex pattern. For example, in antiferromagnetic systems with spin-glass or frustrated magnetization, TRSB can occur in the form of exotic states such as the spin current or in the response to an external magnetic field. In some antiferromagnets, spin configurations can break TRS, even though the material may not show spontaneous macroscopic magnetization. This leads to phenomena such as the topological Hall effect, where the spin texture in the antiferromagnetic phase generates a measurable Hall voltage, providing a signature of TRSB.

Role of Spin-Orbit Coupling in TRSB Effects:

Spin-orbit coupling (SOC) is the interaction between an electron's spin and its orbital motion, and it plays a crucial role in enabling TRSB in many materials. The presence of SOC leads to the coupling between spin and momentum, influencing how spins interact within the material. In systems with strong SOC, TRSB is often enhanced due to the spin-momentum locking, which causes a directional asymmetry in the system's electronic structure.

In ferromagnetic materials, spin-orbit coupling can induce effects such as the spin Hall effect (SHE) or the Rashba effect, both of which break time-reversal symmetry. The SHE, in particular, occurs when a charge current flowing through a material with strong SOC generates a transverse spin current, leading to a polarization of electron spins in a direction perpendicular to the current flow. The spin Hall effect results in the creation of spin currents that are sensitive to the presence of TRSB. In addition, spin-orbit torque (SOT) is another phenomenon where SOC plays a role in manipulating the magnetization of ferromagnetic layers, an essential effect in spintronic devices like magnetic random-access memory (MRAM).

In antiferromagnetic systems, spin-orbit coupling can induce non-collinear spin configurations, which are crucial for the emergence of TRSB effects. For instance, in topological insulators and heavy-metal/antiferromagnet heterostructures, SOC enables the observation of the topological Hall effect, where a non-trivial spin texture, such as skyrmions or spin vortices, generates a Hall signal that is sensitive to time-reversal symmetry breaking. These skyrmions, which are stable magnetic configurations, can only exist when TRSB is present, with SOC providing the necessary interactions to stabilize them.

In summary, the role of spin-orbit coupling in TRSB effects is fundamental in magnetic materials, as it enhances and stabilizes TRSB phenomena, contributing to the development of novel spintronic devices. The combination of spintronics, TRSB, and SOC is paving the way for the creation of materials and technologies that harness the spin degree of freedom for new types of memory, logic, and quantum devices. Understanding these interactions is essential for advancing our knowledge of magnetism and for furthering the development of next-generation materials.

4. Topological Insulators and TRSB:

Influence of TRSB on Topological Surface States:

Topological insulators (TIs) are materials that exhibit insulating bulk properties but have conducting surface states protected by time-reversal symmetry (TRS). These surface states, often referred to as "topologically protected states," arise due to the spin-orbit interaction and exhibit remarkable robustness against non-magnetic impurities or disorder. However, when time-reversal symmetry is broken (TRSB), the topological surface states undergo significant transformations.

TRSB can lead to the gapping of the topological surface states, altering their conductive properties. In particular, TRSB can cause the surface states to become massive or localized, thus destroying their topologically protected nature. For example, in systems where ferromagnetic order is introduced on the surface of a topological insulator, the spin-polarized magnetization breaks TRS, which can result in the opening of a gap in the Dirac cone of the surface states. This gap modifies the transport properties of the surface states, potentially leading to the formation of new exotic phases such as the quantum anomalous Hall effect (QAHE), where the system exhibits a quantized Hall conductance without the need for an external magnetic field. These phenomena are particularly interesting for potential applications in quantum computing and spintronics, where TRSB can be engineered to create robust topologically non-trivial states.

TRSB in the Context of Topological Insulators and Quantum Hall Effects:

In the quantum Hall effect (QHE), TRSB is crucial in the formation of the quantized Hall conductivity in two-dimensional electron gases under strong magnetic fields. While the QHE typically occurs due to an applied external magnetic field, the role of TRSB is particularly relevant in systems with intrinsic topological properties, such as topological insulators. The interplay between the topologically protected surface states of TIs and the breaking of time-reversal symmetry can lead to exotic phases like the quantum anomalous Hall effect (QAHE), where a gap is induced at the Fermi level without the need for an external magnetic field.

TRSB in topological insulators manifests when a ferromagnetic layer or magnetic impurity is introduced to break TRS locally. This leads to the opening of a gap in the surface states and the emergence of dissipationless edge currents. In certain two-dimensional systems with topological surface states, TRSB results in a phase transition that gives rise to a quantized Hall conductance even in the absence of an external magnetic field, as observed in the QAHE. The quantum Hall effect in the context of TRSB thus provides an example of how symmetry breaking can give rise to novel electronic phases with topologically protected edge states, which are useful for low-power, high-speed electronic devices.

Symmetry Breaking in Two-Dimensional Electron Gases:

Two-dimensional electron gases (2DEGs), such as those found at the interface of topological insulators or in heterostructures, provide an ideal platform for exploring the effects of TRSB. In a 2DEG, the electron motion is restricted to two dimensions, and when subject to a magnetic field or spin-orbit coupling, the system can exhibit a variety of quantum phenomena, including the quantum Hall effect and spin Hall effect. TRSB can have a profound impact on the behavior of these systems.

In 2DEGs, symmetry breaking can occur due to the introduction of an external magnetic field, ferromagnetic layers, or through the application of spin-orbit coupling. When TRSB is induced in these systems, the spin-polarized current may flow in a direction that is asymmetrical with respect to time-reversal, leading to dissipationless transport. This is particularly relevant in materials such as graphene or topological insulators, where 2DEGs are a natural feature. For example, the introduction of a ferromagnetic material on the surface of a topological insulator can break TRS, causing the surface states to become gapped and inducing an anomalous Hall effect. Similarly, in systems where strong spin-orbit coupling is present, TRSB can stabilize topological defects such as skyrmions or create new phases with non-trivial topological properties, enhancing the control over electronic and spin-based transport.

In summary, the influence of TRSB on topological insulators and two-dimensional electron gases is profound and complex. While TRSB can break the robustness of topologically protected surface states, it also gives rise to new and exciting quantum phases such as the quantum anomalous Hall

effect and spin-polarized transport. The careful engineering of TRSB in these systems opens up possibilities for novel electronic devices and quantum technologies, where symmetry breaking can be harnessed to manipulate and control the properties of topological materials for advanced applications.

5. Detection Methods of TRSB:

Detecting time-reversal symmetry breaking (TRSB) in condensed matter systems is a complex and critical task, as it helps to uncover the underlying quantum states that govern material properties such as magnetism, superconductivity, and topological phases. Several experimental techniques have been developed to probe TRSB, each offering unique insights into different physical aspects of the systems under study. These methods can be broadly categorized into direct and non-invasive techniques, each with its strengths and challenges.

Experimental Techniques for Detecting TRSB

Neutron Scattering:

Neutron scattering is one of the most powerful experimental techniques for studying the magnetic properties of condensed matter systems, including the detection of TRSB. Neutrons interact strongly with magnetic moments due to their spin, making neutron scattering particularly effective in probing the spin configurations of a material. By using polarized neutrons, it is possible to selectively measure the spin alignment in materials and observe changes that result from TRSB.

When time-reversal symmetry is broken in a system, neutron scattering experiments can reveal spontaneous magnetization or anomalous spin textures. In particular, the polarization of scattered neutrons can change depending on whether or not TRS is preserved. For example, in materials with ferromagnetic or antiferromagnetic ordering, neutron scattering can detect the emergence of a net magnetization or any unconventional spin structure that signifies TRSB. Neutron diffraction can also be used to study the interactions between spins and crystal lattice, providing a clear signal of symmetry breaking.

Muon Spin Resonance (μ SR):

Muon spin resonance (μ SR) is another crucial technique for detecting TRSB in condensed matter systems, particularly in superconductors and magnetic materials. μ SR involves implanting positive muons into a sample, which then undergo spin precession in the magnetic field of the material. By measuring the time evolution of the muon spin polarization, scientists can gain detailed information about the local magnetic environment and detect any TRSB-related changes in the material's magnetic properties.

When TRSB is present, the muon spin polarization may exhibit a behavior distinct from what would occur if time-reversal symmetry were intact. For example, in systems exhibiting the quantum anomalous Hall effect (QAHE), μ SR can reveal the formation of topologically protected edge states or the emergence of a non-zero magnetization. The technique is particularly valuable for understanding the dynamics of localized spins and the response of magnetic moments under different conditions.

Non-invasive Detection Methods:

Kerr Effect:

The Kerr effect is a non-invasive optical technique used to probe the magnetization and symmetry properties of materials, including the detection of TRSB. When polarized light is reflected from a magnetic surface, the plane of polarization of the light rotates due to the interaction between the light's electric field and the material's magnetization. This phenomenon, known as the Kerr rotation, is sensitive to the orientation of the magnetization and can indicate whether TRSB is present in a system.

In materials where TRSB occurs, such as in topological insulators with ferromagnetic order, the Kerr rotation is observed to be asymmetric with respect to time reversal. This asymmetric response is a direct signature of TRSB, as it indicates a preferential direction of magnetization. The Kerr effect is widely used in spintronics and magnetism to study domain structures, spin dynamics, and magnetization reversal processes. It allows for real-time, non-invasive measurement of TRSB in magnetic materials and thin films.

Faraday Rotation:

Faraday rotation is another non-invasive technique used to detect TRSB, particularly in materials with optical activity or in systems where the magnetization is aligned along a specific axis. Faraday rotation occurs when polarized light passes through a material in the presence of a magnetic field, causing the plane of polarization to rotate proportionally to the strength of the magnetic field along the propagation direction of the light. This effect is due to the interaction of the light's electric field with the material's magnetization.

The Faraday rotation can be measured as a function of the magnetic field applied, and its asymmetry with respect to the direction of the field provides valuable insight into whether TRSB is present. In cases where TRSB is induced, the Faraday rotation will exhibit a directional preference, further indicating that the system exhibits broken symmetry. This technique is especially useful in studying optically transparent materials or thin films, where direct magnetic probes like μ SR or neutron scattering might be challenging to apply.

Recent Advances in Quantum Imaging for TRSB Detection:

Recent advances in quantum imaging techniques have significantly enhanced the ability to detect TRSB with high precision. Quantum imaging leverages quantum mechanical properties, such as superposition and entanglement, to achieve higher sensitivity and resolution than classical methods. These advances are opening up new possibilities for detecting subtle effects of TRSB in condensed matter systems, particularly in systems that are difficult to probe with traditional techniques.

Quantum Sensing:

Quantum sensing methods, which use quantum coherence and entanglement to enhance measurement precision, have emerged as powerful tools for detecting TRSB. For example, quantum sensors based on nitrogen-vacancy (NV) centers in diamond are capable of measuring magnetic fields with high spatial resolution and sensitivity. These sensors can be used to detect the minute magnetic field variations that arise due to TRSB, providing a direct measure of the breaking of time-reversal symmetry at the nanoscale.

Quantum Interferometry:

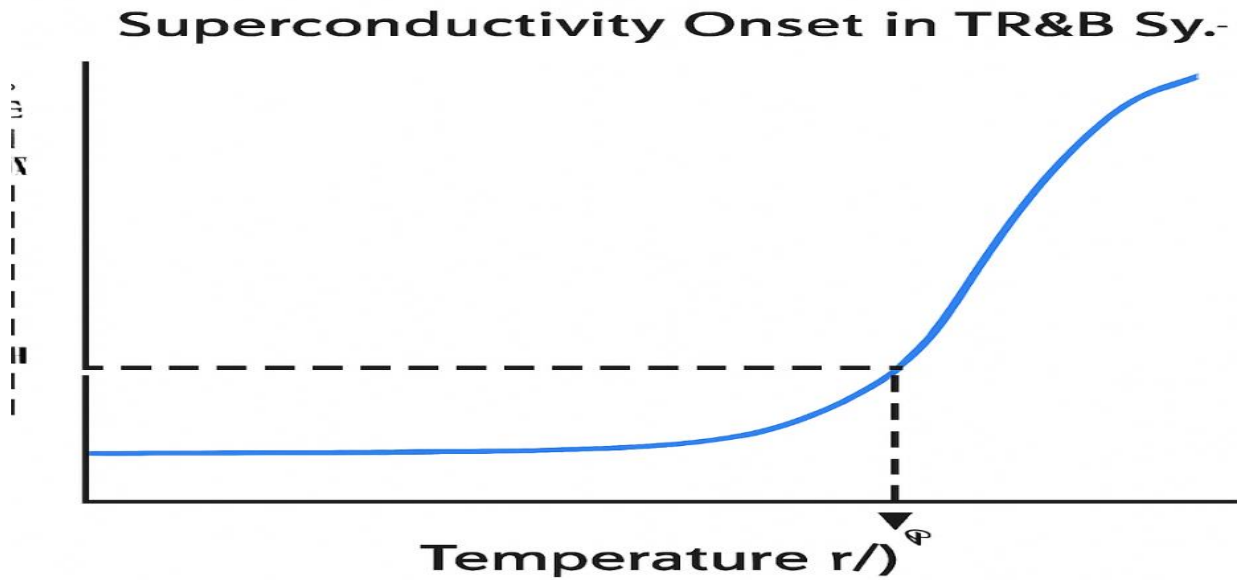
Quantum interferometry, such as the use of Ramsey interferometry, can also be applied to detect TRSB effects in quantum materials. By exploiting the interference between quantum states, these techniques allow for the detection of extremely small changes in the phase of the system's wavefunction, which can be indicative of symmetry breaking. In particular, quantum interferometers can detect changes in the spin or charge density distribution in materials with topologically protected surface states, offering new insights into TRSB in systems like topological insulators or spintronic devices.

Superconducting Qubits and TRSB:

Superconducting qubits, which are a key technology in quantum computing, can also serve as sensitive detectors for TRSB. The coherence times of qubits can be affected by TRSB, and through precise control and measurement of qubit states, it is possible to detect subtle symmetry-breaking effects. In particular, systems where Majorana fermions are predicted to appear, such as

topological superconductors, are ideal candidates for quantum imaging methods that can probe the presence of TRSB.

In conclusion, the detection of TRSB is a crucial aspect of understanding condensed matter systems and advancing spintronic and quantum technologies. Both traditional methods like neutron scattering and μ SR, as well as non-invasive optical techniques like the Kerr effect and Faraday rotation, provide valuable tools for probing TRSB. Recent advances in quantum imaging and sensing offer even more sensitive and precise methods for detecting TRSB in complex materials, paving the way for future studies and applications in quantum physics and materials science.



Summary:

Time-reversal symmetry breaking plays a critical role in the development of novel quantum materials and the understanding of exotic condensed matter phases. This paper has outlined the fundamental concepts of TRSB, with a focus on its occurrence in superconducting, magnetic, and topological systems. TRSB leads to the emergence of new quantum states that are pivotal for advancing our understanding of quantum materials and for developing future technologies in spintronics and quantum computing. The article also highlights the importance of precise experimental methods for detecting TRSB and presents ongoing research in this field.

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